

# Flavor Alignment in SUSY GUTs\*

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## Abstract

A Supersymmetric Grand unified model is constructed based on  $SO(10) \times SO(10)$  symmetry in which new types of Yukawa matrices couple standard and exotic fermions. Evolution of these couplings from the Grand Unified scale to the electroweak scale causes some of them to be driven to their fixed points. This solves the supersymmetric alignment problem and ensures that there are no observable flavor changing neutral currents mediated by supersymmetric particles. Fermion hierarchy and neutrino mixing constraints are automatically satisfied in this formalism.

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# 1 Introduction

Supersymmetry (SUSY), an as yet undetected but highly promising additional symmetry of nature, offers a possible solution to the hierarchy problem [1]. SUSY still has a tremendous amount of theoretical freedom in constructing models: the over one hundred free parameters in the theory must be fixed. Models of supersymmetry breaking can be invoked to reduce the number of arbitrary parameters. In gauge mediated models [2, 3] the pattern of SUSY breaking parameters leads to a low energy theory that has no dangerous flavor changing neutral currents (FCNCs). Models based on gravity mediation of SUSY breaking [4], reduce parameter space considerably, but have no compelling reasons for the absence of flavor changing neutral currents.

It is possible that some of the low energy parameters are determined independent of their values at the Grand Unified Theory (GUT) scale [5, 6]. This happens if the renormalization group equations (RGEs), governing the evolution of the parameters of the theory as one lowers the energy scale, drive some of the parameters to their fixed points. The fundamental values at the GUT scale are then irrelevant. A dynamic reduction of parameter space in this manner automatically solves the FCNC constraint [7], also known as the “SUSY Flavor Alignment” problem.

In this paper, we propose a new model which is “complete” in the sense of being both SUSY and a GUT and illustrates how the idea might work in practice. We revive the Pati-Salam Left-Right idea [8] in an  $SO(10)_L \times SO(10)_R$  framework, enlarging the particle spectrum of the MSSM to include an extra three generations of exotic particles, too heavy to be seen at the electroweak scale. As pointed out previously [7], it is not possible for the standard Yukawas coupling ordinary matter to be close to their fixed points at the weak scale: the fixed point solutions are all  $O(1)$ , yet  $m_u \ll m_t$ , implying a large hierarchy of Yukawa couplings. The main function of the exotics is to provide Yukawa couplings to ordinary (super)matter which do run to their fixed points, leaving the usual Yukawas free. In addition, these exotics provide a natural setting for a seesaw effect [9] in the mass matrices, establishing the observed mass hierarchies in the quark and lepton sectors.

The layout of this paper is as follows: in Section 2 we review the SUSY Flavor Alignment problem and its resolution via the RGE fixed-point running; Section 3 introduces the specifics of the model we propose, including the symmetry-breaking structure, the particle representations, and the couplings that emerge from the superpotential; we discuss the vacuum structure of the theory in Section 4 and demonstrate how the gauge coupling constants evolve from the GUT scale to the weak scale; in Section 5 we verify fermion mass hierarchies and mixings and make some general comments regarding signatures of this model in future experiments. Section 6 summarizes the main conclusions of this paper.

## 2 SUSY Flavor Alignment

The MSSM for 3 generations of matter, represented by the superpotential

$$\mathbf{W} = \mathbf{y}_u \overline{U} Q H_2 + \mathbf{y}_d \overline{D} Q H_1 + \mathbf{y}_e \overline{E} L H_1 + \mu H_1 H_2 \quad (1)$$

with the chiral matter superfields for quarks and leptons  $\overline{U}$ ,  $\overline{D}$ ,  $Q$ ,  $\overline{E}$ ,  $L$ , and Higgs  $H_{1,2}$  charged under  $SU(3)_C \times SU(2)_L \times U(1)_Y$  assigns the 3x3 Yukawa matrices  $\mathbf{y}_{e,d,u}$  to the coupling between particles of different generations. If supersymmetry were an unbroken symmetry of nature, then, after the Higgs fields obtain VEVs,  $O(3)$  rotations in generation space on the quark superfields diagonalize the mass terms and gives rise to the usual CKM matrix in the (s)quark-(s)quark gauge interactions; experiments would report the same CKM matrix for quarks and squarks. However,

since SUSY is broken at an energy  $M_{SUSY}$ , the theory admits generic soft-breaking Yukawa terms in the Lagrangian below this scale:

$$\mathcal{L} \supset (\mathbf{a}_u \bar{U} Q H_2 + \mathbf{a}_d \bar{D} Q H_1 + \mathbf{a}_e \bar{E} L H_1)_{scalar} + (M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B}) \\ + (\sum_{X_i=Q,\bar{u},\bar{d},\bar{e}} X_i^\dagger \mathbf{m}_1^2 X_i + \sum_{i=1,2} m_{H_i}^2 H_i^* H_i + b\mu H_1 H_2)_{scalar} + h.c. \quad (2)$$

where  $(\tilde{g}, \tilde{W}, \tilde{B})$  are the superpartners of the Standard Model gauge bosons and where *scalar* implies that only the scalar component of each superfield is used. Note that the SUSY-breaking parameters  $\mathbf{a}_{u,d,e}$  and  $\mathbf{m}_1^2$  are matrices with *a priori* unknown structure. The squarks have therefore additional sources of inter-generational mixing beyond that for the quarks. At the EW scale the quarks' and squarks' mass terms are not in general diagonalizable by the same rotations and we have separate ‘‘CKM’’ matrices. Experiments such as neutral  $K$ -meson mixing constrain the squark-CKM matrix to be a fraction of a percent deviant from the quark CKM [10]. Since such a high degree of alignment between independently chosen matrices is unlikely, the MSSM is by itself insufficient to naturally account for low-energy flavor structure; this is termed the ‘SUSY Alignment Problem’.

The simplest solution to this problem is to impose a universality on the soft-breaking terms, forcing the SUSY-breaking Yukawas  $\mathbf{a}_{e,d,u}$  to be proportional to the corresponding SUSY Yukawas  $\mathbf{y}_{e,d,u}$ . This is useful for computational purposes yet it lacks any physical motivation. Some models of SUSY breaking such as gauge-mediation [2, 3] have automatic flavor alignment.

There is however an elegant solution [7]: the alignment arises inevitably from the RGEs running to their fixed points. The reason why this occurs is simple: first instead of the explicit soft-breaking terms in the Lagrangian as in (2), we can redefine SUSY-breaking terms as spurions [11] which get  $\theta$ -space dependent vacuum expectation values (VEVs). These VEVs are interpreted as parameters in the superpotential and gauge couplings after making the substitutions:

$$g^2 \rightarrow g^2(1 + M\theta^2 + M\bar{\theta}^2 + 2M^2\theta^2\bar{\theta}^2) \\ \mathbf{y}^{ijk} \rightarrow \mathbf{y}^{ijk} - \mathbf{a}^{ijk}\theta^2 + \frac{1}{2}(\mathbf{y}^{njk}\mathbf{m}_n^{2i} + \mathbf{y}^{ink}\mathbf{m}_n^{2j} + \mathbf{y}^{ijn}\mathbf{m}_n^{2k})\theta^2\bar{\theta}^2 \quad (3)$$

Here the  $\theta^2\bar{\theta}^2$ -dependent terms contribute to the effective action starting at the 1-loop level. Now let the Yukawa couplings all run down to their fixed points; since the RGEs are gauge-dominated, we expect the fixed point solutions for the Yukawas to have a structure that is only a function of the gauge couplings. In particular, each fixed-point solution should factorize into a 3x3 matrix of constants and a gauge-dependent piece since gauge interactions do not depend on flavor:

$$\mathbf{y}_{f.p.} = \mathbf{C} \times \psi(g_i^2) \quad (4)$$

When we switch to the basis where the Yukawa matrix above is diagonal, performing rotations  $Q \rightarrow \Omega Q$  on the left-handed quark fields and separate rotations on each right-handed field, we diagonalize  $\mathbf{y}_{f.p.}\mathbf{y}_{f.p.}^\dagger$  as well, obtaining from (4)

$$\mathbf{y}'_{f.p.}\mathbf{y}'_{f.p.}{}^\dagger = \mathbf{C}' \times \psi(g_i^2) \quad (5)$$

where  $\mathbf{y}'_{f.p.}\mathbf{y}'_{f.p.}{}^\dagger = \Omega\mathbf{y}_{f.p.}\mathbf{y}_{f.p.}^\dagger\Omega^\dagger$  and  $\mathbf{C}' = \Omega\mathbf{C}\Omega^\dagger$ , both of which are diagonal in flavor space. By using the symmetry-breaking rule (3), we can evaluate  $\mathbf{y}\mathbf{y}^\dagger$  after SUSY breaking and quark-field rotation at the fixed point:

$$\mathbf{y}_{f.p.}\mathbf{y}_{f.p.}^\dagger \rightarrow \mathbf{y}'_{f.p.}\mathbf{y}'_{f.p.}{}^\dagger - \mathbf{a}'_{f.p.}\mathbf{y}'_{f.p.}{}^\dagger\theta^2 + O(\bar{\theta}) \quad (6)$$

As (5) and (6) must agree at each order in  $\theta$ , we obtain

$$\mathbf{a}'_{f.p.} \mathbf{y}'^\dagger_{f.p.} = -\mathbf{C}' \times \int \psi(g_i^2) d^2\theta \quad (7)$$

By design  $\mathbf{C}'$  and  $\mathbf{y}'$  are diagonal, and so  $\mathbf{a}'$  is too (likewise one may check  $\mathbf{y}\mathbf{m}^2$ ): SUSY breaking matrices are simultaneously diagonalizable with SUSY Yukawas on the fixed point. This solves the SUSY alignment problem if the amount of RGE running is sufficient to let the fixed-point properties emerge near the electro-weak scale. In this scenario the overwhelming arbitrariness of the soft-SUSY breaking matrices vanishes:  $\mathbf{a}$  and  $\mathbf{m}^2$  are proportional to  $\mathbf{y}$ , the proportionality constant being some function of the various coefficients in the RGEs.

However, since the RGEs' fixed points are functions of  $O(1)$  coefficients we would expect the Yukawa fixed points to all be of order unity, giving a nearly degenerate mass spectrum. Since huge mass hierarchies exist at the EW scale ( $m_t/m_u \approx 10^4$  [12]), the simplest implementation does not work. Taking a cue from [7], we propose a model to address this.

### 3 The $SO(10)_L \times SO(10)_R$ Framework

A model with only MSSM fields may be immediately ruled out because the Yukawa couplings are much too small to have reached  $O(1)$  fixed points. The simplest variant is to let the alignment mechanism discussed above proceed through exotic Yukawas running. Schematically the total Yukawa structure in the low energy Lagrangian would appear as:

$$\mathcal{L} \supset (\mathbf{f} \ \mathbf{f}_\mathbf{H}) \left( \frac{\mathbf{y}v_L}{\mathbf{y}_2v_2} \middle| \frac{\mathbf{y}_1v_1}{\mathbf{Y}\Lambda} \right) \begin{pmatrix} \mathbf{f}^c \\ \mathbf{f}_\mathbf{H}^c \end{pmatrix} \quad (8)$$

Here  $\mathbf{f}, \mathbf{f}_\mathbf{H}$  denote vectors of standard and exotic fermions, respectively, which need not share the same dimensionality for the general mechanism to operate. The VEV of the Higgs coupling to  $\mathbf{y}$  is essentially fixed to be  $v_L \approx m_W$  from experiment, but the magnitudes of the other VEVs  $v_1, v_2, \Lambda$  are free parameters. Provided  $\mathbf{y}_{1,2}$  run to their fixed points an alignment can be obtained. A suitable hierarchy among the VEVs,  $v_L \ll v_1, v_2 \ll \Lambda$  may provide a seesaw solution of the fermion hierarchy problem. A possible hierarchy of VEVs puts the usual  $O(m_W)$  in the  $\mathbf{y}$  corner,  $O(10 \text{ TeV})$  VEVs in the  $\mathbf{y}_\mathbf{x}$  entries, and perhaps VEVs as large as  $O(10^{17} \text{ GeV})$  in the  $\mathbf{Y}$  corner.

Possibly the most elegant realization of (8) involves a left-right symmetry which, upon breaking, yields three generations of exotic  $SU(2)_L$  singlets which couple to themselves through  $\mathbf{Y}$  and to MSSM fields through  $\mathbf{y}_\mathbf{x}$  [13]. The naive breaking pattern  $SU(5)_L \times SU(5)_R \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1) \rightarrow (\text{standard model})$  cannot readily accomodate the low-energy values of the gauge couplings:  $\sin^2\theta_W$  invariably turns out too small [14].

To address this failure we suggest that the low-energy  $SU(3) \times U(1)$  color-electromagnetic

symmetry is the survivor of the following breaking chain:

$$\begin{aligned}
& SO(10)_L \times SO(10)_R \\
& \quad \downarrow \mathbf{M}_{\text{GUT}} \\
& SU(5)_L \times U(1)'_L \times SU(5)_R \times U(1)'_R \\
& \quad \downarrow \mathbf{M}_L \\
& SU(3)_L \times SU(2)_L \times U(1)_L \times SU(5)_R \\
& \quad \downarrow \mathbf{M}_R \\
& SU(3)_L \times SU(2)_L \times U(1)_L \times SU(3)_R \times SU(2)_R \times U(1)_R \\
& \quad \downarrow \mathbf{\Lambda}_{\text{LR}} \\
& SU(3)_C \times U(1)_{L+R} \times SU(2)_L \times SU(2)_R \\
& \quad \downarrow \mathbf{v}_R \\
& SU(3)_C \times U(1)_Y \times SU(2)_L \\
& \quad \downarrow \mathbf{v}_L \\
& SU(3)_C \times U(1)_{EM}
\end{aligned} \tag{9}$$

with the appropriate energy scales noted at the breaking points. In this scheme the scales in the Yukawa matrix (8) correspond as  $v_L \sim \mathbf{v}_L$ ,  $v_{1,2} \sim \mathbf{v}_{L,R}$ , and  $\Lambda \sim \mathbf{\Lambda}_{\text{LR}}$ . The scale of SUSY breaking cannot be much larger than 1 TeV as we assume SUSY solves the hierarchy problem.

All of the standard model quarks and leptons are unified at the GUT scale in three generations of  $SO(10)_L \times SO(10)_R$  spinor representations

$$\{ \chi_L(\mathbf{16}; \mathbf{1}) \oplus \chi_R(\mathbf{1}; \overline{\mathbf{16}}) \} \times 3 \tag{10}$$

The Higgs particles necessary for each stage of the symmetry breaking scheme (9) are contained in the bispinor and tensor representations

$$\begin{aligned}
& \overline{\Phi}(\mathbf{16}; \overline{\mathbf{16}}) \oplus \Phi(\overline{\mathbf{16}}; \mathbf{16}) \\
& \Delta_L(\mathbf{10}; \mathbf{1}) \oplus \Delta_R(\mathbf{1}; \overline{\mathbf{10}}) \\
& \Delta'_L(\mathbf{10}; \mathbf{1}) \oplus \Delta'_R(\mathbf{1}; \overline{\mathbf{10}}) \\
& \Delta''_L(\mathbf{10}; \mathbf{1}) \oplus \Delta''_R(\mathbf{1}; \overline{\mathbf{10}}) \\
& \Theta_L(\mathbf{45}; \mathbf{1}) \oplus \Theta_R(\mathbf{1}; \mathbf{45}) \\
& \Theta'_L(\mathbf{45}; \mathbf{1}) \oplus \Theta'_R(\mathbf{1}; \mathbf{45}) \\
& \quad \oplus \Theta(\mathbf{45}; \mathbf{45})
\end{aligned} \tag{11}$$

These choices of representations are not unique, but they are the minimal set necessary to avoid fine-tuning in the superpotential, as we demonstrate later.

Whereas the Higgs sector is quite flexible in this model, the choice of three matter generations in (10) is more or less fixed, being tightly constrained by searches for a fourth neutrino at collider energies [12]. For ease of discussion in the following, we list in Table 1 (See Appendix) the types of particles present at each stage in the symmetry chain (9).

First we note how the standard model fermions are embedded in the  $SO(10)$  rep's: each 16 of  $SO(10)$  decomposes into a  $1 \oplus \overline{5} \oplus 10$  of  $SU(5)$ . Each collection of 16 states, as in conventional  $SO(10)$  theories [15], represents quarks  $U$  and  $D$  (12 states) and leptons  $L$  and  $E$  (3 states) plus a right-handed neutrino superfield,  $N$  (1 state). Let us show  $\chi_L \oplus \chi_R$ , for example, in more suggestive

$SU(5)$  language:

$$\begin{aligned}
\psi_L &= \begin{pmatrix} \bar{d}_H \\ \bar{d}_H \\ \bar{d}_H \\ e_L \\ -\nu_L \end{pmatrix} & \psi_R &= \begin{pmatrix} d_H \\ d_H \\ d_H \\ \bar{e}_R \\ -\bar{\nu}_R \end{pmatrix} \\
\Psi_L &= \begin{pmatrix} 0 & \bar{u}_H & -\bar{u}_H & -u & -d \\ -\bar{u}_H & 0 & \bar{u}_H & -u & -d \\ \bar{u}_H & -\bar{u}_H & 0 & -u & -d \\ u & u & u & 0 & -\bar{e}_H \\ d & d & d & \bar{e}_H & 0 \end{pmatrix} & \Psi_R &= \begin{pmatrix} 0 & u_H & -u_H & -\bar{u}_R & -\bar{d}_R \\ -u_H & 0 & u_H & -\bar{u}_R & -\bar{d}_R \\ u_H & -u_H & 0 & -\bar{u}_R & -\bar{d}_R \\ \bar{u}_R & -\bar{u}_R & -\bar{u}_R & 0 & -e_H \\ -\bar{d}_R & -\bar{d}_R & -\bar{d}_R & e_H & 0 \end{pmatrix} \quad (12) \\
N_L & & N_R &
\end{aligned}$$

All of the fields with an  $H$ -subscript will acquire masses of order  $\Lambda_{LR}$ ; they are “exotic” particles which are  $SU(2)_L$  singlets at the weak scale. The neutrinos  $N_{L,R}$  are likewise heavy, having masses  $O(M_{GUT})$ . The other particles in (12) form one standard model generation, including a right-handed neutrino,  $\nu_R$ . Note that the ‘mirror symmetry’ is not the usual one between standard fields and exotics [16].

$SO(10)_L \times SO(10)_R$  breaks to  $SU(5)_L \times U(1)'_L \times SU(5)_R \times U(1)'_R$  by the singlet components of the Higgs sector (the  $(\mathbf{1}; \mathbf{1})$  pieces of  $\Phi, \bar{\Phi}$ ) acquiring a VEV. The  $SU(5)_{L,R}$  symmetries are broken by the  $SU(5)$  adjoint fields contained in  $\Theta_{L,R}, \Theta'_{L,R}$  and  $\Theta$  when they get VEVs of the type:

$$\langle \Theta_x \rangle = i \sigma_2 \times \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & b \end{pmatrix} \quad (13)$$

where the particular values of  $a, b$  will depend on  $x \in \{L, L', R, R'\}$ : for the breaking of  $SU(5)_{L,R}$ ,  $a \sim M_{L,R}$ , whereas  $b$  may be zero or non-zero (see the vacuum structure discussion below in 4). The Dimopoulos-Wilczek mechanism [17] for splitting doublets from triplets requires no fine-tuning among the VEVs and parameters in the superpotential and ensures that the colored components ( $i = 1, 2, 3$ ) of all the  $SU(5)$  fundamental and anti-fundamental Higgs ( $H_{xi}, \phi_i$ , see Table 1) get masses  $O(M_{L,R})$  whereas the  $b$ ’s are chosen to make the weak components of  $H_L, \bar{H}_L, H_R, \bar{H}_R, \phi_L, \phi_R$  remain light.

When the  $SU(5)_R$  symmetry breaks, it combines with the  $SU(3)_L$  to form a vector  $SU(3)_{L+R}$  which remains unbroken all the way down to low energies; this we interpret as color. At the same time, the  $U(1)$ ’s in the theory combine to form a vector  $U(1)_{L+R}$ . This vector  $U(1)$  is  $U(1)_{B-L}$  for the standard model fields as is evident from the charge assignments in Table 1; the exotic particles which are absent in the low-energy regime have different charges. The above breaking to  $SU(3)_C \times U(1)$  is achieved by VEVs of the fields  $\omega, \bar{\omega}, \Omega, \bar{\Omega}$  at  $O(\Lambda_{LR})$ ; these themselves acquire masses and decouple from the theory, leaving the uncolored components of  $H_L, \bar{H}_L, H_R, \bar{H}_R$  (all from the  $\Delta_{L,R}$ ) and  $\phi_{L,R}$  (from  $\Phi$ ) as light degrees of freedom. After the breaking of  $SU(2)_R$ , the  $SU(2)_L$  symmetry breaks as in the MSSM with  $H_L, \bar{H}_L$  serving as  $H_{1,2}$  in (1).

### 3.1 Some Minimal Requirements

The relative sizes of the VEVs in (9) are not completely arbitrary. Simple theoretical and phenomenological considerations allow us to put constraints on the magnitudes of  $v_L, v_R, \Lambda_{LR}, M_L, M_R$ , and  $M_{GUT}$ . First of all there must be enough RGE running for the Yukawas to reach fixed points and for the alignment to work. Since it is crucial that the Yukawas  $\mathbf{y}_{1,2}$  in (8) which mix exotic and standard generations run to their fixed points, we must ensure that the running, roughly between  $\Lambda_{LR}$  and  $M_{GUT}$ , is large enough. Of course in order for the RGEs themselves to remain predictive, we must also ensure that no coupling goes nonperturbative (*i.e.*  $g \geq 1$ ) in this regime; typically this forces the GUT scale itself to respect an upper bound just below the Planck scale. There are also important phenomenological constraints which all GUTs must satisfy: masses of new gauge bosons and proton decay. That no experiment has yet detected a  $W_R$ , or right-handed version of the  $W$ -boson, sets a lower bound on its mass of about 1 TeV [18, 19]. Since we expect  $m_{W_R} \approx v_R$ , we must have  $v_R > 1$  TeV. As for proton decay in this model, the usual intermediate gauge boson channels are closed: left- and right-handed quarks are embedded in completely different  $SU(5)$  representations. Proton decay can proceed through colored Higgs' exchange; specifically,  $qqql$  operators arise from the exchange of  $H_x, \phi_x$ . However these operators will be suppressed by the masses of the colored components of these fields which are of order  $M_L$  and  $M_R$ . These masses can actually be as low as  $10^{10}$  GeV [20] and still satisfy the proton lifetime constraint  $\tau_p > 5.5 \times 10^{32} yr$  [21].

Finally, for the exotic fields to have escaped detection, we impose the constraints on the mass of a fourth generation quark [12], giving  $\Lambda_{LR} \geq 200$  GeV. This is a lower bound, but in fact we will find it advantageous in Section 4 to consider much larger  $\Lambda_{LR}$ .

Altogether, we may list the various phenomenological constraints on the VEVs:

$$\begin{aligned} M_{GUT} &\leq 10^{18} GeV \\ M_{GUT}/v_R &\geq 10^{10} \\ M_{L,R} &\geq 10^{10} GeV \\ \Lambda_{LR}, v_R &\geq 10^3 GeV \\ v_L &\approx 10^2 GeV \end{aligned} \tag{14}$$

### 3.2 Superpotential and Yukawa Structure

One way to make the Dimopoulos-Wilczek mechanism operate effectively [17] is to impose a  $Z_3$  symmetry on the superpotential at the GUT scale with charges

$$\begin{aligned} 0 : & \quad \{\chi_{L,R} \ \Delta_{L,R} \ \Theta \ \Phi \ \overline{\Phi}\} \\ 1 : & \quad \Theta_{L,R} \\ 2 : & \quad \{\Delta'_{L,R} \ \Delta''_{L,R} \ \Theta'_{L,R}\} \end{aligned} \tag{15}$$

The most general  $SO(10)_L \times SO(10)_R$  superpotential is then

$$W = W_Y + W_H \tag{16}$$

where

$$\begin{aligned} W_Y &= \lambda_1 \chi_L \chi_R \Phi + \lambda_2 \chi_L^2 \Delta_L + \lambda_3 \chi_R^2 \Delta_R \\ W_H &= \lambda_4 \Phi \overline{\Phi} + \lambda_5 \Phi \overline{\Phi} \Theta + \lambda_6 \Delta_L \Theta_L \Delta'_L + \lambda_7 \Delta_L \Theta_L \Delta''_L \\ &\quad + \lambda_8 \Delta'_L \Theta'_L \Delta''_L + \lambda_9 \Delta_R \Theta_R \Delta'_R + \lambda_{10} \Delta_R \Theta_R \Delta''_R + \lambda_{11} \Delta'_R \Theta'_R \Delta''_R \\ &\quad + \lambda_{12} \Theta^2 + \lambda_{13} \Theta_L \Theta'_L + \lambda_{14} \Theta_R \Theta'_R \end{aligned} \tag{17}$$

In the above the  $\Delta'_{L,R}, \Delta''_{L,R}$  provide a coupling which splits the doublets from the triplets in the physical  $\Delta_{L,R}$  fields which propagate at low energies ( $\Delta_L^2 \Theta_L$  for example vanishes by the antisymmetry of the **45**). The  $\Delta'_{L,R}, \Delta''_{L,R}$  are otherwise inert in the theory as we can arrange for them to have masses of  $O(M_{L,R})$  taking  $a, b \sim M_{L,R}$  as discussed earlier (see (13) ff.).

Under  $SU(5)_L \times SU(5)_R \times U(1)^2$ , the fields decompose as follows:

$$\begin{aligned}
\chi_L &\longrightarrow \psi_L \oplus \Psi_L \oplus N_L \\
\chi_R &\longrightarrow \psi_R \oplus \Psi_R \oplus N_R \\
\Delta_L &\longrightarrow H_L \oplus \overline{H}_L \\
\Delta_R &\longrightarrow H_R \oplus \overline{H}_R \\
\Delta'_L &\longrightarrow H_{L'} \oplus \overline{H}_{L'} \\
\Delta'_R &\longrightarrow H_{R'} \oplus \overline{H}_{R'} \\
\Delta''_L &\longrightarrow H_{L''} \oplus \overline{H}_{L''} \\
\Delta''_R &\longrightarrow H_{R''} \oplus \overline{H}_{R''} \\
\Phi &\longrightarrow \phi_0 \oplus \phi_L \oplus \phi_R \oplus \omega \oplus \Omega \oplus \sigma_1 \oplus \sigma_2 \oplus \sigma_3 \oplus \sigma_4 \\
\overline{\Phi} &\longrightarrow \overline{\phi}_0 \oplus \overline{\phi}_L \oplus \overline{\phi}_R \oplus \overline{\omega} \oplus \overline{\Omega} \oplus \overline{\sigma}_1 \oplus \overline{\sigma}_2 \oplus \overline{\sigma}_3 \oplus \overline{\sigma}_4 \\
\Theta_L &\longrightarrow \Theta_{L1} \oplus \Theta_{L2} \oplus \Theta_{L3} \oplus \Sigma_L \\
&\vdots \\
&\vdots \\
&\vdots
\end{aligned} \tag{18}$$

in accord with the branching rules  $\mathbf{16} \longrightarrow \overline{\mathbf{5}} \oplus \mathbf{10} \oplus \mathbf{1}$ ,  $\mathbf{10} \longrightarrow \mathbf{5} \oplus \overline{\mathbf{5}}$ ,  $\mathbf{45} \longrightarrow \mathbf{1} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{24}$ , and their conjugates.

In this notation, the effective Yukawa terms just below  $M_{GUT}$  in (17) become

$$\begin{aligned}
W_Y = & \lambda_1(\psi_L \psi_R \omega + \Psi_L \Psi_R \Omega + f(\sigma_i)) \\
& + \lambda_2(\psi_L \Psi_L \overline{H}_L + \Psi_L \Psi_L \overline{H}_L) \\
& + \lambda_3(\psi_R \Psi_R \overline{H}_R + \Psi_R \Psi_R \overline{H}_R)
\end{aligned} \tag{19}$$

Group theory indices are implicit;  $\Psi_L \Psi_R \Omega$  means  $\Psi_L^{\alpha\beta} \Psi_{R\alpha'\beta'} \Omega_{\alpha\beta}^{\alpha'\beta'}$ , for example. Generation indices on  $\lambda_{1,2,3}$  are likewise hereafter suppressed. The first two  $\lambda_1$ -terms give large masses to the exotic particles after  $\omega$  and  $\Omega$  get VEVs. Note in particular that the exotic neutrinos  $N_{L,R}$  get a GUT-scale mass given by the VEV of the  $SU(5)_L \times SU(5)_R$  singlet  $\phi_0$ . Terms dependent on the  $\sigma_i$ -fields will not be phenomenologically relevant in the remainder of this study; we choose the vacuum structure of the theory (discussed in Section 4 below) to guarantee this.

The  $\lambda_{2,3}$ -terms of (19) play the role of the couplings  $\mathbf{y}_x$  in (8) responsible for driving the alignment in the RGEs, with the uncolored components of the  $H_{L,R}$  fields getting VEVs at  $v_{L,R}$  respectively, *e.g.*  $\lambda_2 \Psi_L \Psi_L \overline{H}_L$  mixes  $\overline{u}_H$  and  $u$ . Because the right-handed sector mixes the opposite chirality combination, *e.g.*  $\lambda_3 \Psi_R \Psi_R \overline{H}_R$  mixes  $\overline{u}$  and  $u_H$ , we see that  $\mathbf{y}_1 v_1 \neq \mathbf{y}_2 v_2$  in general and one of  $v_{1,2}$  is necessarily  $O(m_W)$ . As long as  $\mathbf{y} \ll \mathbf{y}_{1,2}$  the alignment will work; small standard Yukawas  $\mathbf{y}$  are guaranteed in this model both by symmetry and the choice of vacuum (see (23) below).

## 4 The Model: Vacuum Structure



#### 4.1 Minimizing the Scalar Potential

The scalar potential receives contributions from the superpotential

$$W_H = \begin{aligned} & \lambda_4 \Phi \bar{\Phi} + \lambda_5 \Phi \bar{\Phi} \Theta + \lambda_6 \Delta_L \Theta_L \Delta'_L + \lambda_7 \Delta_L \Theta_L \Delta''_L \\ & + \lambda_8 \Delta'_L \Theta'_L \Delta''_L + \lambda_9 \Delta_R \Theta_R \Delta'_R + \lambda_{10} \Delta_R \Theta_R \Delta''_R + \lambda_{11} \Delta'_R \Theta'_R \Delta''_R \\ & + \lambda_{12} \Theta^2 + \lambda_{13} \Theta_L \Theta'_L + \lambda_{14} \Theta_R \Theta'_R \end{aligned} \quad (20)$$

For this choice of representations the D-term contributions to the scalar potential from  $\Delta$ 's and  $\Phi, \bar{\Phi}$  vanish in the limit where the VEVs from each conjugate pair of fields are equal, whereas those for  $\Theta$ 's vanish according to their symmetry structure (13). Because we have a theory which remains supersymmetric all the way down past  $v_R$ , the minimization of the effective potential implies

$$\frac{\partial W}{\partial \eta} = 0 \quad \eta \supset \{\text{scalars}\} \quad (21)$$

Since in (20) the  $SO(10)^2$  symmetry is already broken by the  $(SU(5) \times U(1))^2$  singlet fields  $\phi_0, \bar{\phi}_0$ , these fields are effectively non-propagating below  $M_{GUT}$ . The next symmetries to break are  $SU(5)_{L,R}$ : as noted earlier in Section 3, the  $\Theta, \Theta_{L,R}$ , and  $\Theta'_{L,R}$  fields get diagonal VEVs as in (13), breaking the  $SU(5)$ s to the subgroup structure  $SU(3) \times SU(2) \times U(1)$  and splitting the colored triplets from the doublets in the  $H$ 's and  $\phi$ 's. We will obtain four sets of light ( $O(v_{L,R})$ ) doublets  $H_L, \bar{H}_L, H_R, \bar{H}_R, \phi_{L,R}, \bar{\phi}_{L,R}$ , four sets of heavy doublets  $H_{L',L''}, \bar{H}_{L',L''}, H_{R',R''}, \bar{H}_{R',R''}$ , and all color triplets heavy for the VEV structure

$$\begin{aligned} \langle \Theta_{L,R} \rangle &= i \sigma_2 \times \begin{pmatrix} M_{L,R} & 0 & 0 & 0 & 0 \\ 0 & M_{L,R} & 0 & 0 & 0 \\ 0 & 0 & M_{L,R} & 0 & 0 \\ 0 & 0 & 0 & v_{L,R} & 0 \\ 0 & 0 & 0 & 0 & v_{L,R} \end{pmatrix} \\ \langle \Theta'_{L,R} \rangle, \langle \Theta \rangle &= i \sigma_2 \times \begin{pmatrix} M_{L,R} & 0 & 0 & 0 & 0 \\ 0 & M_{L,R} & 0 & 0 & 0 \\ 0 & 0 & M_{L,R} & 0 & 0 \\ 0 & 0 & 0 & M_{L,R} & 0 \\ 0 & 0 & 0 & 0 & M_{L,R} \end{pmatrix} \end{aligned} \quad (22)$$

Below  $M_R$  we find the following VEV structure accommodates a minimum:

$$\langle \omega \rangle = \begin{pmatrix} \Lambda_{LR} & 0 & 0 & 0 & 0 \\ 0 & \Lambda_{LR} & 0 & 0 & 0 \\ 0 & 0 & \Lambda_{LR} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \langle \bar{\omega} \rangle = \begin{pmatrix} \Lambda_{LR} & 0 & 0 & 0 & 0 \\ 0 & \Lambda_{LR} & 0 & 0 & 0 \\ 0 & 0 & \Lambda_{LR} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (23)$$

$$\begin{aligned} \langle \Omega_{\alpha\beta}^{\alpha'\beta'} \rangle &= \langle \bar{\Omega}_{\alpha\beta}^{\alpha'\beta'} \rangle = \Lambda_{LR} \quad \text{for} \\ &\begin{pmatrix} \alpha' & \beta' \\ \alpha & \beta \end{pmatrix} \subset \left( \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix} \right) \\ &= 0 \quad \text{otherwise} \end{aligned} \quad (24)$$

In the above it should be remembered that  $\langle \Omega_{\alpha\beta}^{\alpha'\beta'} \rangle = - \langle \Omega_{\alpha\beta}^{\beta'\alpha'} \rangle = \langle \Omega_{\beta\alpha}^{\beta'\alpha'} \rangle$  (and likewise for  $\langle \bar{\Omega} \rangle$ ) since both  $\Omega$  and  $\bar{\Omega}$  are antisymmetric  $SU(5) \times SU(5)$  tensors. Also, factors of  $O(1)$  may

multiply the above VEVs without upsetting the minimization. There is a vacuum degeneracy but we assume the particular vacuum in (24). Besides breaking the theory to a vector  $SU(3) \times U(1)$ , the VEVs above give masses to the exotic fields  $(u_H, d_H, e_H)$  of order  $\Lambda_{LR}$ . This is a crucial ingredient of the seesaw mechanism which we discuss in more detail in Section 5.

Since SUSY is assumed to be broken below  $v_R$  we must include the SUSY-breaking terms when breaking  $SU(2)_L$ . This is a standard exercise in the MSSM, where the scalar potential takes the form

$$V = (|\mu|^2 + m_{H_1}^2)|H_1^0|^2 + (|\mu|^2 + m_{H_2}^2)|H_2^0|^2 - bH_1^0 H_2^0 - b^* H_1^{0*} H_2^{0*} + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2 \quad (25)$$

We ensure that a similar potential arises in our model at  $v_L$ : simply rename  $H_{1,2}$  to  $H_L, \overline{H}_L$  in (25). The gauge couplings  $g, g'$  are understood as usual to be the coupling constants of  $SU(2)_L \times U(1)_Y$ . The low energy spectrum will match the MSSM except now FCNCs are naturally suppressed and a mass hierarchy is automatic (as discussed in Section 5).

## 4.2 Beta Functions and Running Couplings

Since we are working with a GUT, we require that the couplings unify at  $M_{GUT}$ . Each VEV is associated with a threshold in the RGEs where the group symmetry and number of propagating particles change, so the running serves as an indirect constraint on the values of  $v_L, v_R, \Lambda_{LR}, M_L, M_R$ , and  $M_{GUT}$ . We use 1-loop  $\beta$ -functions at each stage of the supersymmetric theory where the symmetry group takes the form  $G \times G_1 \times \dots \times G_n$ :

$$\beta = \frac{g^3}{16\pi^2} \left( \sum_i C_2(R_i) d_1(R_i) \dots d_n(R_i) - 3C_1(G) \right) + o(g^5) \quad (26)$$

Here  $C_2(R_i)$  is the index of the irreducible representation  $R_i$ , defined as

$$C_2(R) \delta^{ab} \equiv \text{Tr}[\mathbf{T}_R^a \mathbf{T}_R^b] \quad (27)$$

for generators  $\mathbf{T}_R$  in the representation  $R$ ,  $C_1(G)$  is  $C_2(R_{adj})$  with  $R_{adj}$  the adjoint rep of  $G$ , and  $d_i(R_j)$  is the dimension of the representation  $R_i$  in the algebra  $G_i$ . We use the version of (26) with broken SUSY at energies below 1 TeV:

$$\beta_{n.S.} = \frac{g^3}{16\pi^2} \left( \frac{2}{3} \sum_f C_2(R_f) d_1(R_f) \dots d_n(R_f) + \frac{1}{3} \sum_s C_2(R_s) d_1(R_s) \dots d_n(R_s) - \frac{11}{3} C_1(G) \right) + o(g^5) \quad (28)$$

with now separate sums over fermionic and scalar rep's. At each characteristic energy scale, the number of particle rep's to account for in (26) or (28) changes: below  $M_{L,R}$ , the  $\Sigma$ 's,  $\sigma$ 's,  $\Delta$ 's and  $\Delta''$ 's decouple; below  $\Lambda$ , the  $\omega, \overline{\omega}, \Omega$ , and  $\overline{\Omega}$  decouple; finally below  $v_R$ ,  $\phi_R, \overline{\phi}_R, H_R, \overline{H}_R$  decouple. The detailed derivation of the  $\beta$ -functions above for the present model appear in the Appendix. Here we present the result of a sample running in Figure 1 below, with the choices of parameters

$$\begin{aligned} \alpha_{GUT}^{-1} &= 10 \\ M_{GUT} &\geq 10^{16} \text{ GeV} \\ M_L &= 10^{16} \text{ GeV} \\ M_R &= 10^{15} \text{ GeV} \\ \Lambda_{LR} &= 5 \cdot 10^{12} \text{ GeV} \\ v_R &= 5 \text{ TeV} \end{aligned} \quad (29)$$

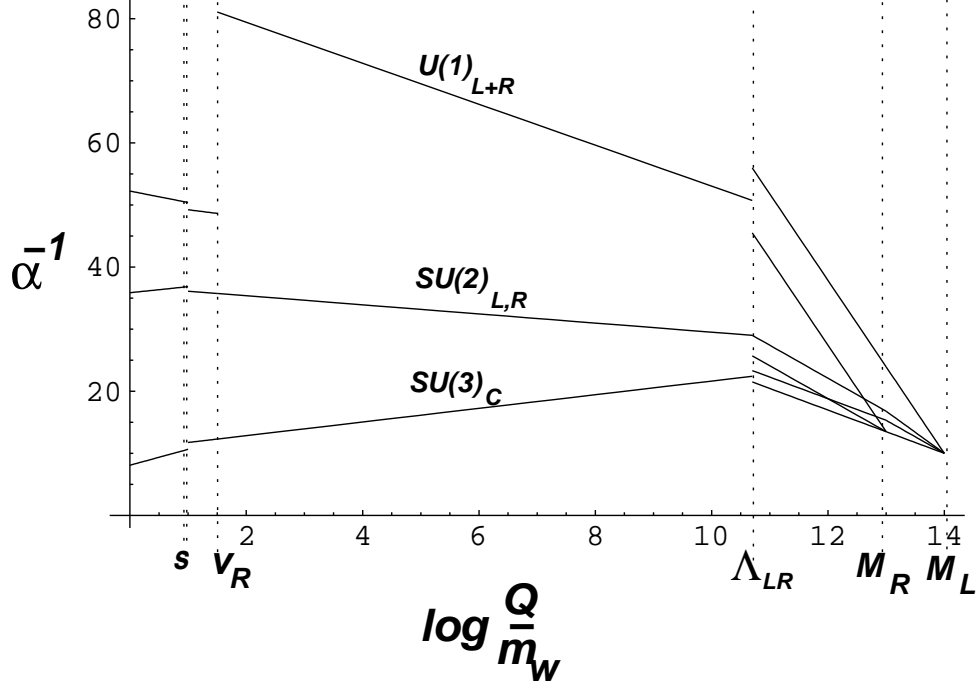


Figure 1: One-Loop Running of the Coupling Constants *with the choice of parameters in (29)*. In the interval  $(M_R, M_L)$ , the runnings are for (top to bottom)  $U(1)_L$ ,  $SU(2)_L$ ,  $SU(3)_L$ , and  $SU(5)_R$ ; just above  $\Lambda_{LR}$ , the couplings are for (top to bottom)  $U(1)_L$ ,  $U(1)_R$ ,  $SU(2)_L$ ,  $SU(2)_R$ ,  $SU(3)_L$ , and  $SU(3)_R$ . The runnings are computed with non-SUSY  $\beta$ -functions below the scale  $s \sim 1$  TeV. Threshold effects are neglected.

These choices satisfy the constraints (14) and are not fine-tuned. From the figure it is clear that the coupling constants at the weak scale are in rough agreement with the experimental values,  $\alpha_s^{-1} = 8.33$ ,  $\alpha_L^{-1} = 29.69$ , and  $\alpha_Y^{-1} = 58.8$  [12]; We expect the agreement to be much better after accounting for particle threshold effects and using two-loop RGEs, as studies of similar models confirm [22]. For the sake of our discussion, however, the qualitative agreement between the weak-scale couplings predicted from this model and the experimental values is sufficient to consider the parameter choices (29) as 'typical'.

### 4.3 Low Energy Fermion Mass Matrices

Putting together the Yukawa couplings in (8), the Higgs' VEVs (23), and the constraints (29), we obtain a seesaw-type mass matrix for the fermions at energies  $< v_L$ . Interestingly, the matrix structure exhibits a dichotomy between quarks/charged leptons and neutrinos. For the quarks and charged leptons the mass matrix takes the general form

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_{H1} \\ f_{H2} \\ f_{H3} \end{pmatrix}^\dagger \left( \begin{array}{ccc|ccc} I & \cdot & \cdot & II & \cdot & \cdot \\ \cdot & o(<< v_L) & \cdot & \cdot & o(v_L)\lambda_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline III & \cdot & \cdot & IV & \cdot & \cdot \\ \cdot & o(v_R)\lambda_3 & \cdot & \cdot & o(\Lambda_{LR})\lambda_1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array} \right) \begin{pmatrix} f_1^c \\ f_2^c \\ f_3^c \\ f_{H1}^c \\ f_{H2}^c \\ f_{H3}^c \end{pmatrix} \quad (30)$$

Here the  $f$ 's represent standard model fermions, whereas the  $f_H$ 's are heavy exotic fields, all of which come in three generations. Strictly speaking, the entries in Quadrant I of (30) are zero in this model, but one can relax this with a VEV structure slightly differing from (23) or with radiative effects. Provided that these entries are small, our results are essentially unchanged. We have noted the matrix structure in the other quadrants in accordance with the notation of the superpotential (19). The exotics  $u_H, d_H$  and  $e_H$  acquire heavy masses in Quadrant IV of  $O(\Lambda_{LR})$ .

The neutrino sector exhibits a completely different mixing structure. In the basis  $\psi_\nu \equiv (\nu_L, \nu_R^c, \mathbf{N}_L, \mathbf{N}_R^c)$  the  $12 \times 12$ -mixing matrix has the form

$$\overline{\psi}_\nu^c \left( \begin{array}{cc|cc} \cdot & \cdot & o(v_L)\lambda_2 & o(v_L)\lambda_1 \\ \cdot & \cdot & o(v_R)\lambda_1 & o(v_R)\lambda_3 \\ \hline o(v_L)\lambda_2 & o(v_R)\lambda_1 & 0 & o(M_{GUT})\lambda_1 \\ o(v_L)\lambda_1 & o(v_R)\lambda_3 & o(M_{GUT})\lambda_1 & 0 \end{array} \right) \psi_\nu^c \quad (31)$$

There are several interesting features of the above structure:

- the seesaw for the neutrinos is more severe than for the quarks and charged leptons, since  $v_L v_R / M_{GUT} \ll v_L v_R / \Lambda_{LR}$ . This results in much smaller masses for neutrinos than for charged leptons. This is in agreement with the tiny mass limits on neutrinos [23].
- the flavor mixing in the leptonic sector can be quite different from that in quark sector
- the physical neutrinos will be Majorana particles

In the next section we will investigate more fully the range of parameters in the above mass matrices which can accomodate the known masses and mixings of the standard generations.

## 5 Constraints and Predictions

### 5.1 Fermion Masses and Neutrino-Mixing

The low energy manifestation of this model is essentially contained in the structure of the mass matrices (30), (31) and the mixing matrices derived from them in the presence of weak interactions. In this section we will demonstrate how the parameters of the model can accomodate the empirical bounds on these two sets of measurements.

The most recent determination of quark masses [12] gives

$$\begin{array}{ll} m_u = 1 \text{ to } 5 \text{ MeV} & m_d = 3 \text{ to } 9 \text{ MeV} \\ m_c = 1.15 \text{ to } 1.35 \text{ GeV} & m_s = 75 \text{ to } 170 \text{ MeV} \\ m_t = 174.3 \pm 5.1 \text{ GeV} & m_b = 4 \text{ to } 4.4 \text{ GeV} \end{array} \quad (32)$$

Our model must be able to replicate these hierarchies (as well as the leptonic ones). Fortunately the seesaw mechanism is a natural feature of the model, as is evident in the structure of (30). For a typical parameter set as in (29), the extreme range of the quark masses in (32) is replaced by two much smaller imposed disparities: first, some  $O(1)$  inhomogeneity of the small Yukawas in Quadrant I of (30), and secondly, some favoring of the heavier quarks to mix more with the extremely heavy exotics. We reserve numerical details for the Appendix, where our results indicate that such mixing may occur at a level sufficient to drive up the heavy quark masses, yet still not contribute to FCNCs, as we now discuss.

Constraints on fermion mixing involve both quarks and neutrinos. Both types of mixing at present have fairly well measured bounds, yet have completely opposite structures: quarks are observed to mix minimally with each other, yet recent experiments seem to suggest that neutrinos prefer to mix in a maximal fashion [24].

Quark mixing is straightforward: upon diagonalizing  $\mathbf{M}\mathbf{M}^\dagger$  (here  $\mathbf{M}$  is the matrix in (30)), *e.g.* for up-quarks, denote the normalized eigenstates as

$$|up\rangle = \sum_i \alpha_i |u_i\rangle + \sum_i \beta_i |u_{H_i}\rangle \quad (33)$$

To obtain a quark eigenstate with mass  $m_q$ , choose

$$\alpha_i \approx 1 \quad \beta_i \geq \frac{v_L \Lambda_{LR}}{v_R^2 + \Lambda_{LR}^2 - m_q^2} \quad (34)$$

In the limit  $\beta_i \rightarrow 0$ , the eigenvalues are dependent solely on the tiny Yukawas and such masses correspond to the lighter quarks  $m_{u,d,s,c}$ . For the heavier quarks  $m_{b,t}$  the  $\beta_i$  of the above magnitudes must be carefully selected taking into consideration the matrix structures in (30). We obtain fits for  $\beta_i$  as high as  $O(10^{-2})$ . This level of mixing with exotic quarks is completely within the bounds set by unitarity of the CKM. Furthermore the measured entries of the CKM matrix can be matched within their error-bars by adjusting the small entries in Quadrant I of the mass matrices (see Appendix).

Neutrino data now seems to favor the Large Mixing Angle (LMA) solution to  $\nu$ -oscillation [24]. If this solution is correct, the lepton-neutrino mixing matrix  $\mathbf{U}_{\text{MNS}}$  (the analog [25] of the CKM matrix for quarks) which mixes  $(e, \mu, \tau)$  with  $(\nu_e, \nu_\mu, \nu_\tau)$  takes the form [26]

$$\mathbf{U}_{\text{MNS}} = \begin{pmatrix} 0.7 & -0.7 & < 0.2 \\ 0.5 & 0.5 & -0.7 \\ 0.5 & 0.5 & 0.7 \end{pmatrix}_{l+\nu} \quad (35)$$

The mass-splitting between the neutrino mass-eigenstates  $(\nu_1, \nu_2, \nu_3)$  is also constrained [26]:

$$\begin{aligned} \Delta m_{32}^2 &\approx 3 \cdot 10^{-3} \text{ eV}^2 \\ \Delta m_{21}^2 &\approx 5 \cdot 10^{-5} \text{ eV}^2 \end{aligned} \quad (36)$$

As in the quark sector, we encounter no difficulty reproducing this data with the given form of the neutrino mass matrix, making a suitable ansatz (see Appendix) for the masses of the observable neutrinos in the model. Note that only a tiny amount of mixing with exotics is necessary to drive the standard neutrino masses to very small values (replace  $\Lambda_{LR}$  with  $M_{GUT}$  and  $m_q$  with  $m_\nu$  in (34)), leaving them the freedom to mix near maximally with each other as in (35). This model thus naturally predicts that mixing among the neutrinos is larger than among the quarks.

## 5.2 Predictions at the Next Experiments

Perhaps the two most testable signatures of this model are the existence of Majorana neutrinos and the appearance of a new vector boson,  $W_R$ , mediating a force between right-handed fermion currents which exactly mirrors the known properties of the Weinberg-Salam weak force. The most sensitive type of experiment at present to test both of these predictions is neutrino-less double- $\beta$  decay, the decay  $N \rightarrow N' + 2e^-$ . The strongest limit at present is for the half life of  $^{76}\text{Ge}$  [27]:

$$\tau_{1/2} > 1.6 \cdot 10^{25} \text{ yr} \quad (37)$$

The decay can proceed through either standard  $W_L$  exchange (suppressed by neutrino helicity flip) or in the present model through the exchange of the  $W_R$  boson or Majorana neutrino. Given the above bound on the lifetime and including present theoretical uncertainties in calculating nuclear physics effects, the limits on the mass  $m_\nu$  of a Majorana neutrino or  $W_R$  [18, 19] are:

$$\begin{aligned} m_\nu &< (few) \text{ eV} \\ m_{W_R} &> 1.6 \text{ TeV} \end{aligned} \quad (38)$$

which tells us that  $v_R/M_{GUT} < 10^{-11}$  and  $v_R \geq 1.6 \text{ TeV}$ , consistent with (29). If a decay is observed in upcoming double- $\beta$  decay experiments [28, 29, 30], we will hopefully have more stringent tests of the current model.

As pointed out in [7], SUSY models which produce flavor alignment through the RGEs generically predict that up-squarks are heavier than down-squarks, in contrast to the usual prediction of, *e.g.*, the minimal supergravity MSSM [4]. This follows simply from matching the  $\theta^2 \bar{\theta}^2$  terms in (5), yielding a condition analogous to (6):

$$\begin{aligned} m_Q^2 + m_u^2 + m_{H_u}^2 &= \xi(g_i) \\ m_Q^2 + m_d^2 + m_{H_d}^2 &= \xi(g_i) \end{aligned} \quad (39)$$

The conditions for EW symmetry breaking,

$$\begin{aligned} |\mu|^2 + m_{H_u}^2 &= b \cot\beta + m_Z^2/2 \cos 2\beta \\ |\mu|^2 + m_{H_d}^2 &= b \tan\beta - m_Z^2/2 \cos 2\beta \end{aligned} \quad (40)$$

and the expression for the mass of the pseudo-scalar Higgs  $A^0$ ,  $m_{A^0}^2 = 2 b/\sin 2\beta$ , give

$$\tilde{m}_u^2 - \tilde{m}_d^2 = -(m_{A^0}^2 + m_Z^2) \cos 2\beta \quad (41)$$

Requiring that the top Yukawa coupling  $y_t$  remain perturbative down to the electroweak scale forces  $\tan\beta > 1$  which implies  $\cos 2\beta < 0$ . Under these assumptions (41) states  $\tilde{m}_u^2 > \tilde{m}_d^2$  as claimed. If Supersymmetry can be found and measured with precision at the next collider experiments [31], (41) is a quantitative prediction.

## 6 Conclusions

In this paper we have constructed a specific model which illustrates a solution to the SUSY flavor problem. The RGEs run to their fixed points, furnishing low energy Yukawa matrices that are independent of their GUT-scale values. It produces the pattern of flavor mixings in the quark sector consistent with experiment. Neutrino mixings and masses are also in agreement with experiment. Flavor violations in the squark sector are small. The unified scale group structure,  $SO(10)_L \times SO(10)_R$ , is an extension of the original Pati-Salam unification ansatz  $SU(4)_L \times SU(4)_R$ , and seems to be the minimal arrangement possible to achieve all of the features derived.

The predictions this model makes at the next accelerator experiments will of course include observation of SUSY particles, but also a right-handed current, possibly at energies as low as  $1 \text{ TeV}$ , analogous to that coupling the  $W$  and  $Z$  particles to standard model doublets. Further, if enough of the the squarks are seen and measured, the squark mass spectrum will have a characteristic pattern of up-squarks being heavier than down-squarks, the opposite of conventional predictions in the MSSM.

## Acknowledgments

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Table 1: Particle Representations at each Energy Scale. *The symmetry group (left column) breaks due to fields (center column) getting VEVs; see (9) for breaking scheme and scales. Prior to the breaking (left column), fields (right column) describe the matter content. Fields which play little phenomenological role, e.g. the  $\sigma_i$ 's in (18), are omitted for brevity but may be determined from the rep's in the first row of the table and  $SO(10)$  branching rules (see discussion following (18))*

Symmetry	Fields getting VEVs	Matter
$SO(10)_L \times SO(10)_R$	$\Phi(\mathbf{16}; \overline{\mathbf{16}}) \quad \overline{\Phi}(\overline{\mathbf{16}}; \mathbf{16})$	$\{\chi_L(\mathbf{16}; \mathbf{1}) \quad \chi_R(\mathbf{1}; \overline{\mathbf{16}})\} \times 3$ $\Delta_L, \Delta'_L, \Delta''_L(\mathbf{10}; \mathbf{1})$ $\Delta_R, \Delta'_R, \Delta''_R(\mathbf{1}; \overline{\mathbf{10}})$ $\Theta_L, \Theta'_L(\mathbf{45}; \mathbf{1})$ $\Theta_R, \Theta'_R(\mathbf{1}; \mathbf{45}) \quad \Theta(\mathbf{45}; \mathbf{45})$
$SU(5)_L \times U(1)'_L$ $\times SU(5)_R \times U(1)'_R$	$\Sigma_L, \Sigma'_L(\mathbf{24}; 0; \mathbf{1}; 0)$ $\Sigma_R, \Sigma'_R(\mathbf{1}; 0; \mathbf{24}; 0)$ $\Sigma_{LR}(\mathbf{24}; 0; \mathbf{24}; 0)$	$\psi_L(\overline{\mathbf{5}}; 3; \mathbf{1}; 0) \quad \psi_R(\mathbf{1}; 0; \mathbf{5}; -3)$ $\Psi_L(\mathbf{10}; -1; \mathbf{1}; 0) \quad \Psi_R(\mathbf{1}; 0; \overline{\mathbf{10}}; 1)$ $\omega(\mathbf{5}; -3; \overline{\mathbf{5}}; 3) \quad \overline{\omega}(\overline{\mathbf{5}}; 3; \mathbf{5}; -3)$ $\Omega(\overline{\mathbf{10}}; 1; \mathbf{10}; -1) \quad \overline{\Omega}(\mathbf{10}; -1; \overline{\mathbf{10}}; 1)$ $\phi_L(\mathbf{5}; -3; \mathbf{1}; 0) \quad \overline{\phi}_L(\overline{\mathbf{5}}; 3; \mathbf{1}; 0)$ $\phi_R(\mathbf{1}; 0; \overline{\mathbf{5}}; 3) \quad \overline{\phi}_R(\mathbf{1}; 0; \mathbf{5}; -3)$ $+ \sigma$ terms (see (18)) $H_L(\mathbf{5}; -3; \mathbf{1}; 0) \quad \overline{H}_L(\overline{\mathbf{5}}; 3; \mathbf{1}; 0)$ $H_R(\mathbf{1}; 0; \overline{\mathbf{5}}; 3) \quad \overline{H}_R(\mathbf{1}; 0; \mathbf{5}; -3)$
$SU(3)_L \times SU(2)_L \times U(1)_L$ $\times SU(3)_R \times SU(2)_R \times U(1)_R$	$\omega \quad \overline{\omega}$ $\Omega \quad \overline{\Omega}$	$q(\mathbf{3}; \mathbf{2}; 1; \mathbf{1}; \mathbf{1}; 0) \quad \overline{u}_H(\overline{\mathbf{3}}; \mathbf{1}; -4; \mathbf{1}; \mathbf{1}; 0)$ $\overline{d}_H(\overline{\mathbf{3}}; \mathbf{1}; -2; \mathbf{1}; \mathbf{1}; 0) \quad l(\mathbf{1}; \mathbf{2}; 3; \mathbf{1}; \mathbf{1}; 0)$ $\overline{e}_H(\mathbf{1}; \mathbf{1}; 6; \mathbf{1}; \mathbf{1}; 0) \quad u_H(\mathbf{1}; \mathbf{1}; 0; \mathbf{3}; \mathbf{1}; 4)$ $\overline{q}(\mathbf{1}; \mathbf{1}; 0; \overline{\mathbf{3}}; \mathbf{2}; -1) \quad \overline{l}(\mathbf{1}; \mathbf{1}; 0; \mathbf{1}; \mathbf{2}; -3)$ $d_H(\mathbf{1}; \mathbf{1}; 0; \mathbf{3}; \mathbf{1}; 2) \quad e_H(\mathbf{1}; \mathbf{1}; 0; \mathbf{1}; \mathbf{1}; -6)$ $\phi_L(\mathbf{1}; \mathbf{2}; 3; \mathbf{1}; \mathbf{1}; 0) \quad \overline{\phi}_L(\mathbf{1}; \mathbf{2}; -3; \mathbf{1}; \mathbf{1}; 0)$ $\phi_R(\mathbf{1}; \mathbf{1}; 0; \mathbf{1}; \mathbf{2}; -3) \quad \overline{\phi}_R(\mathbf{1}; \mathbf{1}; 0; \mathbf{1}; \mathbf{2}; 3)$ $H_L(\mathbf{1}; \mathbf{2}; 3; \mathbf{1}; \mathbf{1}; 0) \quad \overline{H}_L(\mathbf{1}; \mathbf{2}; -3; \mathbf{1}; \mathbf{1}; 0)$ $H_R(\mathbf{1}; \mathbf{1}; 0; \mathbf{1}; \mathbf{2}; -3) \quad \overline{H}_R(\mathbf{1}; \mathbf{1}; 0; \mathbf{1}; \mathbf{2}; 3)$
$SU(3)_C \times U(1)_{L+R}$ $\times SU(2)_L \times SU(2)_R$	$\phi_R \quad \overline{\phi}_R$ $H_R \quad \overline{H}_R$	$q(\mathbf{3}; 1/3; \mathbf{2}; \mathbf{1}) \quad \overline{u}_H(\overline{\mathbf{3}}; -4/3; \mathbf{1}; \mathbf{1})$ $\overline{d}_H(\overline{\mathbf{3}}; 2/3; \mathbf{1}; \mathbf{1}) \quad l(\mathbf{1}; -1; \mathbf{2}; \mathbf{1})$ $\overline{e}_H(\mathbf{1}; 2; \mathbf{1}; \mathbf{1}) \quad u_H(\mathbf{3}; 4/3; \mathbf{1}; \mathbf{1})$ $\overline{q}(\mathbf{3}; -1/3; \mathbf{1}; \mathbf{2}) \quad \overline{l}(\mathbf{1}; \mathbf{1}; \mathbf{1}; \mathbf{2})$ $d_H(\mathbf{3}; -2/3; \mathbf{1}; \mathbf{1}) \quad e_H(\mathbf{1}; -2; \mathbf{1}; \mathbf{1})$ $\phi_L(\mathbf{1}; -1; \mathbf{2}; \mathbf{1}) \quad \overline{\phi}_L(\mathbf{1}; \mathbf{1}; \mathbf{2}; \mathbf{1})$ $H_L(\mathbf{1}; -1; \mathbf{2}; \mathbf{1}) \quad \overline{H}_L(\mathbf{1}; \mathbf{1}; \mathbf{2}; \mathbf{1})$
$SU(3)_C \times U(1)_Y \times SU(2)_L$	$\phi_L \quad \overline{\phi}_L$ $H_L \quad \overline{H}_L$	MSSM
$SU(3)_C \times U(1)_{EM}$		SM

## Appendix

### 1. Particle Representations



Table 2: Group Theory Indices

$G$	$R_i$	$C_2(R_i)$
$SU(5)$	<b>1</b>	0
	<b>5</b>	1/2
	<b>10</b>	3/2
$SU(3)$	<b>1</b>	0
	<b>3</b>	1/2
$SU(2)$	<b>1</b>	0
	<b>2</b>	1/2

## 2. Beta Functions

To calculate the beta function expressions (26) and (28),

$$\beta = \frac{g^3}{16\pi^2} \left( \sum_i C_2(R_i) d_1(R_i) \cdots d_n(R_i) - 3C_1(G) \right) \quad (42)$$

$$\beta_{n.s.} = \frac{g^3}{16\pi^2} \left( \frac{2}{3} \sum_f C_2(R_f) d_1(R_f) \cdots d_n(R_f) + \frac{1}{3} \sum_s C_2(R_s) d_1(R_s) \cdots d_n(R_s) - \frac{11}{3} C_1(G) \right) \quad (43)$$

one needs to know the “indices”  $C_2(R_i)$  ; in Table 2 below we list the indices for various rep’s of  $SU(5)$ ,  $SU(3)$ , and  $SU(2)$  (for a more extensive discussion, see [32]):

If the group under consideration is  $U(1)$ , the index  $C_2(R_i)$  is the sum of the squares of the (normalized)  $U(1)$  charges,  $\sum_i y_i^2$ .

We have the further rule that  $C_1(SU(N)) = N$ . With all of this we can compute the beta functions (see Table 3).

In addition to knowing the beta-functions, one must also use the matching conditions at each threshold where symmetries change. There are two basic rules to follow when the higher energy symmetry  $G_+$  changes to  $G_-$  below the threshold:

- If  $G_- \subset G_+$  , then the matching condition is  $\alpha_- = \alpha_+$
- If the generators of  $G_-$  are linear combinations of the generators of several of the higher energy groups (labelled by  $j$ ),  $\mathbf{T}_- = \sum_j c_j \mathbf{T}_j$ , then the proper matching condition for the couplings is  $\alpha_-^{-1} = \sum_{j,k} c_j^2 \alpha_j^{-1}$

With this we have the following matching conditions:

$$\begin{aligned} \alpha_C^{-1} &= \frac{1}{2}(\alpha_{3L}^{-1} + \alpha_{3R}^{-1}) & at \quad \mu = \Lambda_{LR} \\ \alpha_{1,L+R}^{-1} &= \frac{1}{2}(\alpha_{1L}^{-1} + \alpha_{1R}^{-1}) & at \quad \mu = \Lambda_{LR} \\ \alpha_Y^{-1} &= \frac{3}{5}\alpha_{1R}^{-1} + \frac{2}{5}\alpha_{1,L+R}^{-1} & at \quad \mu = v_R \end{aligned} \quad (44)$$

## 3. Quark and Neutrino Constraints

Let us take the CKM matrix given as

$$\mathbf{V}_{CKM} \approx \begin{pmatrix} 1 & -0.2 & 0.004 \\ 0.2 & 1 & -0.03 \\ 0.002 & 0.04 & 1 \end{pmatrix} \quad (45)$$

Table 3: First Order Beta-Function Coefficients. The notation is defined so  $\frac{dg_G}{d(\ln\mu)} = \frac{\beta_G g_G^3}{16\pi^2}$ . Here  $s \sim 1 \text{ TeV}$  is the scale where *SUSY* breaks.

Energy Range	$G$	$\beta_G$
$M_R < \mu < M_L$	$SU(5)_R$	19/2
	$SU(3)_L$	29/2
	$SU(2)_L$	37/2
	$U(1)_L$	38
$\Lambda_{LR} < \mu < M_R$	$SU(3)_R$	29/2
	$SU(2)_R$	37/2
	$U(1)_R$	38
	$SU(3)_L$	29/2
	$SU(2)_L$	37/2
	$U(1)_L$	38
$v_R < \mu < \Lambda_{LR}$	$SU(3)_C$	-3
	$SU(2)_R$	2
	$SU(2)_L$	2
	$U(1)_{L+R}$	8
$s < \mu < v_R$	$SU(3)_C$	-3
	$SU(2)_R$	2
	$U(1)_Y$	8
$v_L < \mu < s$	$SU(3)_C$	-7
	$SU(2)_R$	-8/3
	$U(1)_Y$	5

where we are neglecting the small  $CP$ -violating pieces, and further make the assumption that the exotic sector mixes in the same way; the total 6 by 6 “extended-CKM” matrix (eCKM) looks like

$$\mathbf{V}_{eCKM} = \left( \begin{array}{c|c} \mathbf{V}_{CKM} & < 10^{-3} \\ \hline < 10^{-3} & \mathbf{V}_{CKM} \end{array} \right) \quad (46)$$

where the off-diagonal entries are bounded from above by unitarity. We assume that the exotic quarks mix in the same proportions as the standard quarks do only for simplicity; for a more precise fit it is necessary to treat the exotic quark mixing as a collection of free parameters.

As in the standard model, one diagonalizes the quark mass matrix (30) by performing unitary transformations on the left- and right-handed quarks:

$$\begin{aligned} u_L &\longrightarrow \mathbf{U}_{\mathbf{u}} u_L & d_L &\longrightarrow \mathbf{U}_{\mathbf{d}} d_L \\ u_R &\longrightarrow \mathbf{V}_{\mathbf{u}} u_R & d_R &\longrightarrow \mathbf{V}_{\mathbf{d}} d_R \end{aligned} \quad (47)$$

Here the  $u$ ’s and  $d$ ’s are 6-vectors of standard and exotic quarks. The phenomenological constraints are

$$\begin{aligned} \mathbf{V}_{eCKM} &= \mathbf{U}_{\mathbf{u}}^\dagger \mathbf{U}_{\mathbf{d}} \\ \lambda_{\mathbf{u}} \lambda_{\mathbf{u}}^\dagger &= \mathbf{U}_{\mathbf{u}} \mathbf{D}_{\mathbf{u}}^2 \mathbf{U}_{\mathbf{u}}^\dagger \\ \lambda_{\mathbf{d}} \lambda_{\mathbf{d}}^\dagger &= \mathbf{U}_{\mathbf{d}} \mathbf{D}_{\mathbf{d}}^2 \mathbf{U}_{\mathbf{d}}^\dagger \end{aligned} \quad (48)$$

with  $D_{u,d}$  being the diagonal quark mass matrices which must accomodate the measured quark masses in (32). There is a great deal of freedom of parameters in satisfying (48): the matrices  $\lambda_{1,2,3}$  as well as the small ( $\ll 1$ ) Yukawas in (30) permit a class of solutions that give flavor-basis mass matrices like

$$\left( \begin{array}{ccc|ccc} I & \cdot & \cdot & II & \cdot & \cdot \\ \cdot & o(10^{-6} - 10^{-5}) & \cdot & \cdot & o(10^{-1} - 10^0) & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline III & \cdot & \cdot & IV & \cdot & \cdot \\ \cdot & o(10^0 - 10^2) & \cdot & \cdot & o(10^8 - 10^{10}) & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array} \right) \times m_W \quad (49)$$

for the up-sector with the ‘typical’ parameters (29) of the model ( $\tan\beta \approx 10$ ), and similarly for the down-sector. The tiny couplings in Quadrant I have a significant effect on the mass eigenvalues; a typical pattern for  $\mathbf{U}_{\mathbf{u}}$  might be <sup>†</sup>

$$\mathbf{U}_{\mathbf{u}} \approx \left( \begin{array}{ccc|ccc} \cdot & \cdot & \cdot & \approx 0 & \approx 0 & \approx 0 \\ \cdot & o(1) & \cdot & < 10^{-3} & < 10^{-3} & < 10^{-3} \\ \cdot & \cdot & \cdot & \approx 10^{-2} & \approx 10^{-2} & \approx 10^{-2} \\ \hline \approx 0 & < 10^{-3} & \approx 10^{-2} & \cdot & \cdot & \cdot \\ \approx 0 & < 10^{-3} & \approx 10^{-2} & \cdot & o(1) & \cdot \\ \approx 0 & < 10^{-3} & \approx 10^{-2} & \cdot & \cdot & \cdot \end{array} \right) \quad (50)$$

The mixing between exotic and standard quarks is tiny, well below unitarity and FCNC bounds, yet sufficient to give sizable masses to the heavier  $c, t$ -quarks since the exotics themselves are so massive. The matrix calculations are therefore rather sensitive to small changes, yet can accomodate the constraints in (48).

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<sup>†</sup>we do not furnish exact numbers here since these will depend upon the choice of  $\mathbf{V}_{eCKM}$  and the  $\mathbf{V}_{\mathbf{u},\mathbf{d}}$ , both of which entail not currently observable physics and therefore may follow as suits the taste of the model-builder

The only extra bit of analysis needed for the neutrino phenomenology is the value of the masses to insert in the analogue of  $\mathbf{D}_\nu$  (the charged lepton masses  $\mathbf{D}_l$  we know fairly well, of course). If the LSND experiment's results are genuine [33], then it may be reasonable to assume an average neutrino mass around 1 eV. The rest of the work closely parallels the above discussion for quarks, with the result that the constraints analogous to (48) for lepton masses and the BNS-matrix may be satisfied within this model.

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